David Trotman

This volume contains the proceedings of the international workshop "Topology and Geometry of Singular Spaces", held in honour of David Trotman in celebration of his sixtieth birthday. The workshop took place at the Centre International de Rencontres Mathématiques (CIRM), Marseilles, France from October 29 to November 2nd 2012. Its main theme was the singularity theory of spaces and maps.

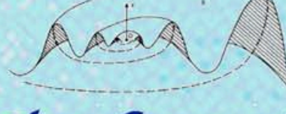
The meeting was attended by 74 participants from all over the world. 29 talks were given by major specialists, and 8 posters were presented by some younger mathematicians. The topics of the talks and posters were wide-ranging: stratification theory, stratified Morse theory, geometry of definable sets, singularities at infinity of polynomial maps, additive invariants of real algebraic varieties, applications of singularities to robotics, and topology of complex analytic singularities.

We thank all participants, especially the speakers, for making the meeting successful and fruitful, both socially and scientifically.

We are also very grateful to all the research bodies who contributed to the financing of the conference: the CIRM institution, the University of Aix-Marseille for Fonds FIR, the LABEX Archimède, and FRUMAM, the University of Rennes 1, the University of Savoy, the ANR SIRE, the city of Marseilles, the "Conseil Gènèral des Bouches du Rhône", the Ministry of Education via the ACCES program, the GDR (Groupement de Recherche) of the CNRS *Singularités et Applications* and the GDR-International *franco-japonais-vietnamien de singularités*.

The papers of this volume cover a variety of the subjects discussed at the workshop. All the manuscipts have been carefully peer-reviewed. We thank the authors for their valuable contributions, and the referees for their careful and conscientious work.

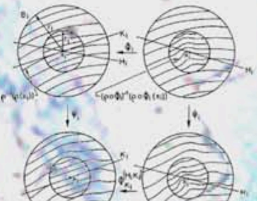
The Editors (who were also the organisers of the workshop),
April 2015.



Geometry and Topology of Singular Spaces

In honour of David Trotman for his 60th birthday

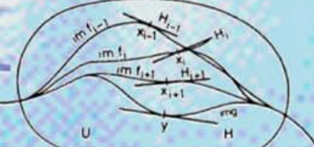
October 29th - November 2nd, 2012

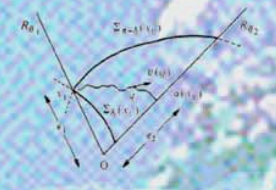


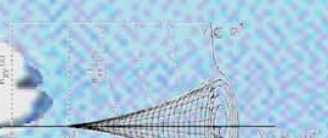
CIRM

Centre International de Rencontres Mathématiques

Luminy - Marseille







Confirmed Speakers

J.-P. Brasselet
H. Brodersen
M. Coste
J. Damon
H. Hamm
S. Koike
K. Kurdyka
Lê Dung Tráng
C. McCrory
A. Parusinski
A. du Plessis
M. Ruas
M. Shiota
B. Teissier
C.T.C. Wall
L. Wilson

Organizing Committee

K. Bekka, ***Université de Rennes I***
N. Dutertre, ***Université d'Aix-Marseille***
C. Murolo, ***Université d'Aix-Marseille***
A. du Plessis, ***University of Aarhus***
S. Simon, ***Université de Savoie***
G. Valette, ***Polska Akademia Nauk***

Scientific Committee

G. Comte, ***Université de Savoie,***
J. Damon, ***University of North Carolina***
P. Orro, ***Université de Savoie***
A. Parusinski, ***Université de Nice***
M. Ruas, ***Universidade de Sao Paolo***
L. Wilson, ***University of Hawaii***
C.T.C. Wall, ***University of Liverpool***

Aix*Marseille université IRMAR PAN ANR CONSEIL GENERAL

David Trotman

David John Angelo Trotman was born on September 27th, 1951, in Plymouth, U.K..

As a boy he lived in Stourbridge, attending first Gig Mill School (1958-62) and then King Edward's School (1962-69). His interest in, and enjoyment of, mathematics was evident very early, but he had many other interests too - the end of his schooldays coincided with success in a competition involving walking 240 kilometers through mountainous terrain in western Turkey, undertaking a dozen set projects on the way!

He read mathematics as an undergraduate at the University of Cambridge (1969-72), where a prize-winning essay on *Plane Algebraic Curves* is already indicative of his mathematical focus. He went on to do post-graduate work at the University of Warwick - from where his M.Sc. dissertation *Classification of elementary catastrophes of codimension less than or equal to 5*, often cited and much used, comes - and later on at the Université Paris-Sud, at Orsay. His Ph.D. was awarded in 1978; his thesis was entitled *Whitney Stratifications: Faults and Detectors*. The list of advisors who encouraged this work is remarkable - Christopher Zeeman, Bernard Teissier, René Thom, and, less officially, Terry Wall.

He has held tenured positions at the University of Paris XI (Orsay) and at the University of Angers, but from 1988 he has been Professor of Mathematics at the University of Provence (Aix-Marseille I). Here he has played an important role, both administratively (for example, he was Director of the Graduate School in Mathematics and Computing of Marseilles from 1996 to 2004, and he was an elected member of the CNU (the National University Council in France) from 1999 to 2007), and especially in teaching and research. He has supervised ten Ph.D. students, with great success - all ten are active in teaching and/or research in mathematics. They are listed below.

David's extensive published research in singularity theory is described by Les Wilson elsewhere in this volume. An equally important part of David's contribution to the research milieu lies in the way he interacts with colleagues and students. He is very helpful, and very generous with his time - and his wide knowledge of, and intuition for, singularities in general and stratifications in particular has helped towards the success of many a research project, to the extent that René Thom, in an article in the Bulletin of the AMS, could write of the work of "Trotman and his school" on the theory of stratifications. David has always been good at asking interesting questions, and at finding, or helping to find, interesting answers!

I have known David since we were undergraduates at St. John's College, Cambridge, and over time have got to know him, and his family, very well. He has helped me very many times, both mathematically and practically. It is a privilege to have him as my colleague and my friend. I congratulate him on a most successful career so far, and wish for him - and for us, his colleagues, collaborators and students - many more years of interesting mathematics.

Andrew du Plessis

The Trotman School of Stratifications.

First generation: Patrice Orro (1984); Karim Bekka (1988); Stephane Simon (1994); Laurent Noirel (1996);, Claudio Murolo (1997); Georges Comte (1998); Didier D'Acunto (2001); Dwi Juniati (2002); Guillaume Valette (2003); Saurabh Trivedi (2013).

The next generation: via Orro: Mohammad Alcheikh, Abdelhak Berrabah, Si Tiep Dinh, Farah Farah, Sébastien Jacquet, Mayada Slayman; via Bekka: Nicolas Dutertre, Vincent Grandjean; via Comte: Lionel Alberti; via Juniati: Mustamin Anggo, M.J. Dewiyani, Sulis Janu, Jackson Mairing, Theresia Nugrahaningsih, Herry Susanto, Nurdin.

The coauthors: Kambouchner, Brodersen, Navarro Aznar, Orro, Bekka, Kuo, Li Pei Xin, Kwieciński, Risler, Wilson, Murolo, Noirel, Comte, Milman, Juniati, du Plessis, Gaffney, King, Plénat, Trivedi and Nguyen Nhan.

THE RESEARCH OF DAVID TROTMAN

Leslie Wilson

In order to analyze singular spaces (differentiable or analytic), Whitney and Thom in the 1950's and 1960's partitioned the spaces into disjoint unions of manifolds satisfying some conditions on how they approached each other; this was the beginning of Stratification Theory. Early work by them, Mather and others focused on proving topological equisingularity of the stratifications, or of stratified mappings. The theory has continued to develop, and has become an essential tool in Differential Topology, Algebraic Geometry and Global Analysis. Since his first publications in 1976, David Trotman has played a central role in Stratification Theory. I will give a brief presentation of his work. Citation numbers refer to the following Publication List. I will assume some familiarity with basic stratification theory on the reader's part; an excellent survey of real stratification theory is [C16].

Whitney's stratification condition (b) and Verdier's (w) were both early on proven to guarantee topological equisingularity. How are these conditions related? (b) is equivalent to (w) in the complex analytic case; (w) implies (b) in the real subanalytic case. The converse is not true: the first semialgebraic example appeared in Trotman [C2], the first algebraic example $y^4 = t^4x + x^3$ was due to Brodersen-Trotman [6]. In the differentiable case neither condition implies the other (the slow spiral satisfies (w) but not (b)).

Wall conjectured that condition (b) (and Whitney's weaker condition (a)) were equivalent to more geometric conditions (b_s) and (a_s): these conditions hold for strata X and Y with Y in the closure of X if for every C^1 tubular neighborhood T of Y (with C^1 projection π to Y and C^1 control function ρ to R with $Y = \rho^{-1}(0)$), $(\pi, \rho)|X$ is a submersion to $Y \times R$ (respectively, $\pi|X$ is a submersion to Y)— here π and ρ are assumed C^1-equivalent to orthogonal projection and the distance squared to Y function, respectively. Trotman in [4] showed that (b_s) implies (b) and (a_s) implies (a) (Thom having established the converse earlier). Trotman's Arcata paper [C5] is still a beautiful though no longer complete listing of known relationships between stratification conditions.

For some applications (for example the classification of topological stable mappings) it is necessary to consider stratification conditions weaker than (b) and (w), but which still guarantee topological equisingularity. One useful such condition is condition (C), introduced by Trotman's student Bekka; this involves replacing ρ with a generalized control function. Bekka and Trotman in [25] (see also [11]) study a notion of "locally-radial (C)-regular spaces": in addition to yielding stratifications which are topologically trivial, the stratifications are locally homeomorphic to a cone on a stratified space such that the rays of the cone have finite length and the volume is locally finite. In [C14], Bekka and Trotman define a notion of "weakly Whitney", which lies between (b)-regular and locally-radial (C)-regular; it has the additional property that the intersection of two weakly Whitney stratified spaces is weakly Whitney (see also [32]).

Condition (a) is weaker then (b), and doesn't imply topological triviality; why is it interesting? Trotman showed in [3] that (a) has the following important property: a locally finite stratification of a closed subset Z of a C^1 manifold M is (a)-regular iff for every C^1 manifold N, $\{f \in C^1(N, M)|f$ is transverse to the strata of $Z\ \}$ is an open set in the Whitney C^1 topology.

An important property for stratification conditions is invariance under transverse intersection. The following was proved in Orro-Trotman [C17]: if (Z, Σ) and (Z', Σ') are Whitney (b)-regular (resp. (a)-regular, resp. (w)-regular) and have transverse intersections in M, then $(Z \cap Z', \Sigma \cap \Sigma')$ is (b)-regular (resp. (a)-regular, resp. (w)-regular) (the (b) case was done earlier, the Orro-Trotman result includes other conditions we haven't looked at).

Similarly one would like to know which conditions are invariant under intersection with generic wings. Suppose X and Y are disjoint C^2 submanifolds of a C^2 manifold M, and $y \in Y \cap \overline{X}$. Suppose E is a regularity condition (like (b)). Then (X, Y) is said to be (E^*)-regular if for all k, $0 \leq k < codY$, there is an open, dense subset of the Grassmannian of codimension k subspaces of T_yM containing T_yY such that if W is a C^2 submanifold of M with $Y \subset W$ near y, and $T_yW \in U^k$, then W is transverse to X near y and $(X \cap W, Y)$ is (E)-regular at y (the W is a *generic wing*). From Navarro Aznar-Trotman [7]: for subanalytic stratifications, $(w) \implies (w^*)$, and if $\dim Y = 1$, $(b) \implies (b^*)$. This property plays an important role in the work of Goresky and MacPherson on existence of stratified Morse functions, and in Teissier's equisingularity results. More recently, it was shown by Juniati-Trotman-Valette [26]: for subanalytic stratifications, $(L) \implies (L^*)$ (where (L) is the condition of Mostowski guaranteeing Lipschitz equisingularity).

Another interesting stratification condition, due originally to Thom, is condition (t). Recall Whitney's example $Z = \{y^2 = t^2x^2 + x^3\}$, which satisfies (a) but not (b). The intersections of Z with planes through 0 transverse to the t-axis have constant topological type. A theorem by Kuo in 1978 states: if (X, Y) is (a)-regular at $y \in Y$ then (h^∞) holds, i.e. the germs at y of intersections $S \cap X$, where S is a C^∞ submanifold transverse to Y at y and $\dim S + \dim Y = \dim M$ (S is called a *direct transversal*) are homeomorphic. Trotman refined Thom's condition to be: (X, Y) is (t^k)-*regular* at $y \in Y$ if every C^k submanifold S transverse to Y at y is transverse to X nearby. He proved the following theorems.

Theorem (Trotman [1]): If Y is semianalytic, then (t^1) is equivalent to (a).

In the above result one needs non-direct transversals. In the results below, we always restrict to direct transversals.

Theorem (Trotman [9]): (t^1) is equivalent to (h^1).

Theorem (Trotman-Wilson [17], following Kuo-Trotman [12] and Kuo-Li-Trotman [13]): For subanalytic strata (t^k) is equivalent to the finiteness of the number of topological types of germs at y of $S \cap X$ for S a C^k transversal to Y $(1 \le k \le \infty)$.

The proofs use the"Grassmann blowup": like the regular blowup, but with lines through y replaced with all linear subspaces through y of dimension equal to the codimension of Y.

Theorem ([12] and [17]): (X, Y) is (t^k)-regular at $0 \in Y$ iff its Grassmann blowup $(\tilde{X}, \tilde{Y})$ is (t^{k-1})-regular at every point of $\tilde{Y}$ $(k \ge 1)$.

A definition of (t^k) is given in [17] so that (t^0) is equivalent to (w). So $(t^1) \implies (h^1)$ follows from blowup and then applying the Verdier Isotopy Theorem.

Consider the Koike-Kucharz example: let $Z = \{x^3 - 3xy^5 + ty^6 = 0\}$, with Y the t-axis and $X = Z - Y$. Then (X, Y) is (t^2), but not (t^1). There are two topological types of germs at 0 of intersections $S \cap X$ where S is a C^2 submanifold transverse to Y at 0. However the number of topological types of such germs for S of class C^1 is uncountable.

Also there is a theory in [17] of (t^{k-}) such that (t^{1-}) is essentially (a) holding for all sequences going to 0 not tangent to Y. The (t^k) and (t^{k-})-conditions were formulated for jets of transversals. The (t^k) and (t^{k-})-conditions were then used to characterize sufficiency of jets of functions, generalizing theorems of Bochnak, Kuo, Lu and others.

In Gaffney-Trotman-Wilson[30] condition (t^k) was expressed in terms of integral closure of modules, giving more algebraic techniques for computations. In the complex analytic case, (t^k) is characterized by the genericity of the multiplicity of a certain submodule.

If a subset Z of $\mathbf{R}^n$ or $\mathbf{C}^n$ contains a submanifold Y, and p is the local orthogonal projection to Y, then the normal cone $C_Y Z$ of Z to Y is the set of limits $t_i(z_i - p(z_i))$, where $z_i \in Z$ converge some $y \in Y$, and t_i is in $\mathbf{R}$ or $\mathbf{C}$ as appropriate.

Theorem (Hironaka in analytic (b) case, Orro-Trotman [22] generalize to smooth $(a) + (r^e)$): a stratification of Z satisfying the above regularity conditions is (npf) (= normally pseudo-flat, i.e. p is an open map), and (n) (= the fibre of the normal cone is the tangent cone of the fibre).

Orro-Trotman [22] show the Theorem fails for (a)-regularity. Trotman-Wilson [28] show that it also fails in the non-polynomial bounded o-minimal category for (b); our example is :
$z = f(x, y) = x - x\ln(y + \sqrt{x^2 + y^2})/\ln(x)$.

The Nash fiber of a singular space X at x is the set of limits of tangent spaces at regular points x_i of X as $x_i \to x$. Kwieciński-Trotman [15] show: every continuum can be realized as the Nash fiber of a Whitney stratified set.

The classical Poincaré-Hopf Theorem equates the index of a vector field with isolated zeros on a smooth compact manifold with the Euler characteristic of the manifold. Trotman (with King) proved a generalization to singular spaces satisfying fairly general stratification conditions; their manuscript has been influential in the field for many years, but has only recently been published in [33].

Finally, Trotman has made several contributions toward the proof of Zariski's Conjecture: the multiplicity of complex analytic hypersurface-germs with isolated singularity is invariant under homeomorphism. C^1-invariance was proven in [C11]; bi-Lipschitz invariance is proved in Risler-Trotman [16]. In Comte-Milman-Trotman [23] it is proven that multiplicity is preserved by homeomorphisms which preserve both $|z|$ and level sets of the moduli of our defining equations. More recently, Plenát and Trotman in [31] prove: if the family $F(z, t) = f(z) + tg_1(z) + t^2g_2(z) + t^3g_3(z) + \dots$ has constant Milnor number at $z = 0$, then $mult(g_r) = mult(f) - r + 1$ for $r \ge 1$.

Publications of David Trotman (October 2014)

Articles in refereed journals :

1. A transversality property weaker than Whitney (a)-regularity, *Bulletin of the London Mathematical Society*, 8 (1976), 225–228.

2. Geometric versions of Whitney regularity, *Mathematical Proceedings of the Cambridge Philosophical Society* 80 (1976), 99–101.

3. Stability of transversality to a stratification implies Whitney (a)-regularity, *Inv. Math.* 50 (1979), 273–277.

4. Geometric versions of Whitney regularity for smooth stratifications, *Annales de l'Ecole Normale Supérieure* 12, 4ème série (1979), 453–463.

5. (with Anne Kambouchner), Whitney (a)-faults which are hard to detect, *Annales de l'Ecole Normale Supérieure* 12, 4ème série (1979), 465–471.

6. (with Hans Brodersen), Whitney (b)-regularity is weaker than Kuo's ratio test for real algebraic stratifications, *Mathematica Scandinavia* 45 (1979), 27–34.

7. (with Vicente Navarro Aznar), Whitney regularity and generic wings, *Annales de l'Institut Fourier, Grenoble* 31 (1981), 87–111.

8. (with Patrice Orro), Sur les fibres de Nash de surfaces à singularités isolées, *Comptes Rendues de l'Académie des Sciences de Paris*, tome 299 (1984), 397–399.

9. Transverse transversals and homeomorphic transversals, *Topology* 24 (1985), 25–39.

10. (with Patrice Orro), On the regular stratifications and conormal structure of subanalytic sets, *Bulletin of the London Mathematical Society* 18 (1986), 185–191.

11. (with Karim Bekka), Propriétés métriques de familles Φ-radiales de sous-variétés différentiables, *Comptes Rendues de l'Académie des Sciences de Paris*, tome 305 (1987), 389–392.

12. (with Tzee-Char Kuo), On (w) and (t^s)-regular stratifications, *Inv. Math.* 92 (1988), 633–643.

13. (with Tzee-Char Kuo and Li Pei Xin), Blowing-up and Whitney (a)-regularity, *Canadian Mathematical Bulletin*, 32 (1989), 482–485.

14. Une version microlocale de la condition (w) de Verdier, *Ann. Inst. Fourier, Grenoble*, 39 (1989), 825–829.

15. (with Michal Kwiecinski), Scribbling continua in R^n and constructing singularities with prescribed Nash fibre and tangent cone, *Topology and its Applications*, 64 (1995), 177–189.

16. (with Jean-Jacques Risler), Bilipschitz invariance of the multiplicity, *Bulletin of the London Mathematical Society* 29 (1997), 200–204.

17. (with Leslie Wilson), Stratifications and finite determinacy, *Proceedings of the London Mathematical Society*, (3) 78 (1999), no. 2, 334–368.

18. (with Claudio Murolo), Semi-differentiable stratified morphisms, *Comptes Rendus de l'Académie des Sciences, Paris*, Série I Math. 329 (1999), no. 2, 147–152.

19. (with Claudio Murolo), Horizontally-C^1 morphisms and Thom's isotopy theorem, *Comptes Rendus de l'Académie des Sciences, Paris*, Série I Math. 330 (2000), no. 8, 707–712.

20. (with Patrice Orro), Cône normal à une stratification régulière, *Seminari Geometria, Università degli Studi Bologna*, 12 (2000), 169–175.

21. (with Claudio Murolo), Relèvements continus contrôlés de champs de vecteurs, *Bulletin des Sciences Mathématiques*, 125, 4 (2001), 253–278.

22. (with Patrice Orro), Cône normal et régularités de Kuo-Verdier, *Bulletin de la Société Mathématique de France*, 130 (2002), 71–85.

23. (with Georges Comte and Pierre Milman), On Zariski's multiplicity problem, *Proceedings of the Amer. Math. Soc.*, 130 (2002), no. 7, 2045-2048.

24. (with Claudio Murolo and Andrew du Plessis), Stratified transversality via isotopy, *Transactions of the Amer. Math. Soc.*, 355 (2003), no. 12, 4881–4900.

25. (with Karim Bekka), On metric properties of stratified sets, *Manuscripta Mathematica*, 111(2003), 71–95.

26. (with Dwi Juniati and Guillaume Valette), Lipschitz stratifications and generic wings, *Journal of the London Math. Soc.*, (2) 68 (2003), 133–147.

27. (with Claudio Murolo and Andrew du Plessis), Stratified transversality via time-dependent vector fields, *J. London Math. Soc. (2)* 71 (2005), no. 2, 516–530.

28. (with Leslie Wilson), (r) does not imply (n) or (npf) for definable sets in non polynomially bounded o-minimal structures, *Singularity theory and its applications (ed. S. Izumiya, G. Ishikawa, H. Tokunaga, I. Shimida, T. Sano)*, Advanced Studies in Pure Mathematics 43, Mathematical Society of Japan (2006), 463-475.

29. (with Claudio Murolo), Semidifferentiabilité et version lisse de la conjecture de fibration de Whitney, *Singularity theory and its applications (ed. S. Izumiya, G. Ishikawa, H. Tokunaga, I. Shimida, T. Sano)*, Advanced Studies in Pure Mathematics 43, Mathematical Society of Japan (2006), 271-309.

30. (with Terence Gaffney and Leslie Wilson), Equisingularity of sections, (t^r) condition, and the integral closure of modules, *Journal of Algebraic Geometry* 18 (2009), no. 4, 651-689.

31. (with Camille Plénat), On the multiplicities of families of complex hypersurface-germs with constant Milnor number, *International Journal of Mathematics* 24 (3) (2013), 1350021.

32. (with Karim Bekka), Weak Whitney regularity and Briançon-Speder examples, *Journal of Singularities* **7** (2013), 88-107.

33. (with Henry King), Poincaré-Hopf theorems for singular spaces, *Proceedings of the London Mathematical Society* 108 (3) (2014), 682-703.

34. (with Saurabh Trivedi), Detecting Thom faults in stratified mappings, *Kodai Mathematical Journal* 37 (2) (2014), 341-354.

35. (with Nhan Nguyen and Saurabh Trivedi), A geometric proof of the existence of definable Whitney stratifications, *Illinois Journal of Mathematics* (2014), to appear.

36. (with Duco van Straten), Weak Whitney regularity implies equimultiplicity for families of singular complex analytic hypersurfaces, submitted.

Articles in acts of conferences or collective works :

C1. (with Christopher Zeeman), Classification of elementary catastrophes of codimension less than or equal to 5, *Structural Stability, the Theory of Catastrophes, and Applications, Proceedings, Seattle 1975 (edited by P.J. Hilton)*, Lecture Notes in Math. 525, Springer, New York, 1976, 263–327.

C2. Counterexamples in stratification theory : two discordant horns, *Real and Complex Singularities, Oslo 1976 (edited by P. Holm)*, Sijthoff and Noordhoff, Alphen aan den Rijn, 1977, 679–686.

C3. Interprétations topologiques des conditions de Whitney, *Journées Singulieres à Dijon 1978, Astérisque* 59-60, 1979, 233–248.

C4. Regular stratifications and sufficiency of jets, *Algebraic Geometry, La Rabida 1981*, Lecture Notes in Mathematics 961, Springer, New York, 1982, 492–500.

C5. Comparing regularity conditions on stratifications, *Proceedings of Symposia in Pure Mathematics, Volume 40, Arcata 1981–Singularities, Part 2*, American Mathematical Society, Providence, Rhode Island, 1983, 575–586.

C6. On the canonical Whitney stratification of real algebraic hypersurfaces, *Séminaire de géométrie algébrique réelle (dirigé par Jean-Jacques Risler)*, tome 1, Publications Mathématiques de l'Université de Paris 7, 1986, 123–152.

C7. On Canny's roadmap algorithm, *Real Days - Symposium in honour of Aldo Andreotti, edited by M. Galbiati*, University of Pisa, 1990, 115–117.

C8. On Canny's roadmap algorithm : orienteering on semialgebraic sets (an application of singularity theory to theoretical robotics), *Proceedings of the 1989 Warwick Singularity Theory Symposium (edited by D. M. Q. Mond and J. Montaldi)*, Springer Lecture Notes, 1991, 320–339.

C9. Blowing-up Thom-Verdier regularity, *June 1991 Workshop on resolution of singularities, edited by M. Galbiati*, University of Pisa, 1993, pp.117–124.

C10. Espaces stratifiés réels, *Stratifications and topology of singular spaces (eds. D. Trotman, L. Wilson)*, Hermann - Travaux en Cours, Paris, vol. 55, 1997, 93–107.

C11. Multiplicity as a C^1 invariant, *Real analytic and algebraic singularities (edited by T. Fukuda, T. Fukui, S. Izumiya and S. Koike)*, Pitman Research Notes in Mathematics, vol. 381, Longman, 1998, 215–221.

C12. Singularités, *Dictionnaire d'Histoire et Philosophie des Sciences, edité par D. Lecourt*, Presses Universitaires de France, Paris, 1999, 866–867.

C13. (with Laurent Noirel), Subanalytic and semialgebraic realisations of abstract stratified sets, *Real Algebraic Geometry and Ordered Structures, Proceedings of the special semester, Louisiana State University, Baton Rouge 1996 (edited by C. Delzell and J. Madden), Contemporary Mathematics* 253, American Mathematical Society, Providence, Rhode Island, 2000, 203–207.

C14. (with Karim Bekka), Weakly Whitney stratified sets, *Real and complex singularities, Sao Carlos 1998 (edited by J.W. Bruce and F. Tari)*, Chapman and Hall/CRC Research Notes in Mathematics 412, Boca Raton, Florida, 2000, 1–15.

C15. (with Dwi Juniati), Determination of Lipschitz stratifications for the surfaces $y^a = z^b x^c + x^d$, *Singularités Franco-Japonaises, Sémin. Congr.*, 10, Soc. Math. France, Paris, 2005, 127–138.

C16. Lectures on real stratification theory, *Singularity Theory, Proceedings of the 2005 Marseille Singularity School and Conference, World Scientific* (2007), 139-155.

C17. (with Patrice Orro), Transverse regular stratifications, *Real and Complex Singularities, edited by M. Manoel, M. C. Romero Fuster and C. T. C. Wall, 10th international workshop, Sao Carlos, Brazil 2008*, London Mathematical Society Lecture Note Series **380**, Cambridge University Press (2010), 298-304.

C18. Bilipschitz equisingularity, *Real and Complex Singularities, edited by M. Manoel, M. C. Romero Fuster and C. T. C. Wall, 10th international workshop, Sao Carlos, Brazil 2008*, London Mathematical Society Lecture Note Series **380**, Cambridge University Press (2010), 338-349.

Edited conference proceedings :

1. (with Leslie Wilson) *Singularities of maps and differential equations*, Hermann - Travaux en Cours, 54, 1997.

2. (with Leslie Wilson) *Stratifications and topology of singular spaces*, Hermann - Travaux en Cours, 55, 1997.

3. (with Denis Chéniot, Nicolas Dutertre, Claudio Murolo and Anne Pichon) *Singularity Theory, Proceedings of the 2005 Marseille Singularity School and Conference, World Scientific*, 2007, 1065 pages.

List of the participants :

AROCA Fuensanta, UNAM, Instituto de Matematicas, Cuernavaca
BASU Saugata, Purdue University, West Lafayette
BEKKA Karim, Université de Rennes 1
BIERSTONE Edward, Fields Institute, Toronto
BODIN Arnaud, Université de Lille 1, Villeneuve d'Ascq
BRASSELET Jean-Paul, Institut Mathématique de Luminy, Marseille
BRODERSEN Hans, University of Oslo
CHACHAPOYAS SIESQUEN Nancy Carolina, Université d'Aix-Marseille
COMTE Georges, Université de Savoie, Le Bourget-du-Lac
CORIPACO HUARCAYA Jorge Alberto , Universidade de Sao Paulo
COSTE Michel, Université de Rennes 1, Rennes
DALBELO Thais Maria, Université d'Aix-Marseille
DAMON James, University of North Carolina
DENKOWSKA Zofia, Université d'Angers
DENKOWSKI Maciej, Jagiellonian University of Krakow
DU PLESSIS Andrew, University of Aarhus
DUTERTRE Nicolas, Université d'Aix-Marseille
EYRAL Christophe, Universidade de Sao Paulo
FELICIANI BARBOSA Grazielle, Universidade Federal de Sao Carlos
FERRAROTTI Massimo, Politecnico di Torino
FICHOU Goulwen, Université de Rennes 1
FUKUI Toshizumi, Saitama University
GALLIGO Andre, Université de Nice
GIACOMONI Julien, Université d'Aix-Marseille
GORYUNOV Victor, University of Liverpool
GRANDJEAN Vincent, CNPq-UFC Fortaleza
GRULHA Nivaldo, Universidade de Sao Paulo
HAMM Helmut, University of Muenster
HERBIG Hans-Christian, Aarhus University
ITENBERG Ilia, Université de Paris 6
IZUMI Shuzo, Kinki University
KOIKE Satoshi, Hyogo University
KRZYZANOWSKA Iwona, Gdansk University
KUCHARZ Wojciech, Jagiellonian University of Krakow
KURDYKA Krzysztof, Université de Savoie
LAPEBIE Julie, Université d'Aix-Marseille
LE Dung Trang, Université d'Aix-Marseille
LE GAL Olivier, Université de Savoie
LUDWIG Ursula, Université de Freiburg
MCCRORY Clint, University of Georgia
MENEGON Aurélio, Universidade de Sao Paulo
MOYA PEREZ Juan Antonio, Universitat de Valencia

MUROLO Claudio, Université d'Aix-Marseille
NGUYEN THI BICH Thuy, Université d'Aix-Marseille
NOWEL Aleksandra, Gdansk University
O'SHEA Donal, New College of Florida
ORRO Patrice, Université de Savoie
PARUSINSKI Adam, Université de Nice
PERLEGA Stefan, University of Vienna
PETROV Peter, Universidade de Sao Paulo
PFLAUM Markus, University of Colorado
PLENAT Camille, Université d'Aix-Marseille
RAIBAUT Michel, Université de Savoie
RANDRIAMBOLOLONA Serge, Université de Savoie
RISLER Jean-Jacques, Université de Paris 6
RIZZIOLLI Eleris, UNESP, Rio Claro, BRESIL
ROND Guillaume, Université d'Aix-Marseille
SAIA Marcelo Jose, Universidade de Sao Paulo
SANCHEZ Luis, Universidade de Sao Paulo
SHIOTA Masahiro, Nagoya University
SIERSMA Dirk, University of Utrecht
SIMON Stéphane, Université de Savoie
SOARES RUAS Maria Aparecida, Universidade de Sao Paulo
SPIVAKOVSKY Mark, Université Paul Sabatier
SZAFRANIEC Zbigniew, Gdansk University
TA Le Loi, University of Dalat
TEISSIER Bernard, Intitut Mathématique de Jussieu
TIBAR Mihai, Université de Lille 1
TRIVEDI Saurabh, Université d'Aix-Marseille
TROTMAN David, Université d'Aix-Marseille
VALETTE Anna, Jagiellonian University of Krakow
VALETTE Guillaume, Polish Academy of Science
WALL Charles C T, University of Liverpool
WILSON Leslie, University of Hawaii

List of the talks :

Transversality and resolution of singularities, Edward BIERSTONE

Sur les stratifications de l'ensemble de Jelonek, Jean-Paul BRASSELET

On sufficiency of jets, Hans BRODERSEN

Additive invariants of definable sets, Georges COMTE

Singularités de robots parallèles, Michel COSTE

Topology of Nonisolated and Non-Complete Intersection Matrix Singularities, Jim DAMON

Stratification, resolution, and stable unfoldings of map-germs on singular varieties, Andrew du Plessis

Equidistribution of the roots of a sparse polynomial system, André GALLIGO

Local order 1 invariants of fronts in $\mathbb{R}^3$, Victor GORYUNOV

Collapsing locus of definable Hausdorff limits, Vincent GRANDJEAN

Functions on real analytic spaces and stratified Morse theory, Helmut HAMM

Sequence Selection Property and Bi-Lipschitz Homeomorphism, Satoshi KOIKE

Reaching generalized critical values of a polynomial, Krzystztof KURDYKA

Le Polyèdre d'effondrement, LE Dung Trang

Lojasiewicz inequality on non compact domains and vanishing components, Olivier LE GAL

The Witten deformation for singular spaces and radial Morse functions, Ursula LUDWIG

The weight filtration for real algebraic varieties, Clint McCRORY

Counting branches of the set of self-intersections of a real analytic germ from $\mathbb{R}^2$ to $\mathbb{R}^3$, Aleksandra NOWEL

Du tumulus au gradient horizontal, Patrice ORRO

A Weight Filtration and Additive Invariants for Real Algebraic Varieties, Adam PARUSINSKI

On the stratification theory of orbit and inertia spaces of proper Lie groupoids, Markus PFLAUM

Curvature of Real Algebraic Varieties, Jean-Jacques RISLER

To avoid Vector Fields in Singularity Theory, Masahiro SHIOTA

Extremal Configurations of Polygonal Linkages, Dirk SIERSMA

Filtrations et gradué associé en géométrie des singularités, Bernard TEISSIER

A characterization of Whitney a-regular complex analytic stratifications, Saurabh TRIVEDI

On the local geometry of definably stratified sets, Guillaume VALETTE

Unfoldings in the flat singularity theory of plane curves, Terry WALL

A Survey on the Trotman works, Les WILSON

List of the posters :

Newton filtrations and non-degenerate ideals in the ring of polynomials,
Jorge Alberto Coripaco Huarcaya

Topology of simple singularities of ruled surfaces in $\mathbb{R}^p$, Grazielle Feliciani Barbosa

On the topology of real analytic maps, Nivaldo de Góes Grulha Júnior

On the boundary of the Milnor fibre of real singularities, Aurelio Menegon Neto

Corank 2 map germs from $\mathbb{R}^2$ to $\mathbb{R}^2$, Juan Antonio Moya Pérez

Formal neighbourhoods in arc spaces, Peter Petrov

Cobordism on maps between $\mathbb{Z}_2$-Witt spaces, Eliris Rizziolli

Affine properties of surfaces in $\mathbb{R}^4$: asymptotic lines, Luis Florial Espinoza Sánchez